ACHARYA INSTITUTE OF TECHNOLOGY Bangalore - 560090

USN 10ME64

Sixth Semester B.E. Degree Examination, Dec.2016/Jan.2017 Finite Element Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, selecting atleast TWO questions from each part.

PART - A

1 a. Explain the basic steps involved in FEM.

(06 Marks)

b. Explain briefly about node location system and node numbering scheme.

(08 Marks)

- c. Explain plane stress and plane strain problem with examples and write the relation between stress and strain. (06 Marks)
- 2 a. Derive the stiffness matrix of bar element using direct approach.

(05 Marks)

- b. Using Rayleigh Ritz method, determine the deflection of a cantilever beam subjected to point load at its end. (10 Marks)
- c. Determine the displacement at nodes of spring system shown Fig. Q2(c) using principle of minimum potential energy, (05 Marks)

 $K_1 = 40 \text{ N/mm}$; $K_2 = 60 \text{N/mm}$; $K_3 = 80 \text{ N/mm}$; $F_1 = 60 \text{ N}$ $F_2 = 50 \text{ N}$.

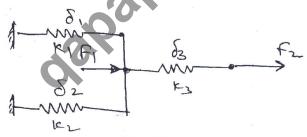


Fig. Q2(c)

- 3 a. Explain simplex, complex and multiplex elements using element shapes. (06 Marks)
 - b. Find the shape functions at point P for the CST element shown in Fig. Q3(b). Also find the area and Jacobian matrix for the element. (08 Marks)

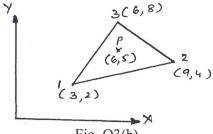


Fig. Q3(b)

c. What are the convergence requirements that an isoperimetric element should satisfy? Sketch and explain 2D Pascal triangle. (06 Marks)

4 a. Obtain the element stresses of the stepped bar shown Fig. Q4(a), take E = 200 GPA. $A_1 = 400 \text{ mm}^2$; $L_1 = 200 \text{ mm}$; $A_2 = 300 \text{ mm}^2$; $L_2 = 150 \text{ mm}$. (10 Marks)

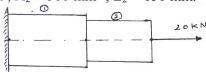
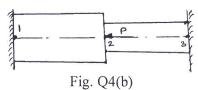


Fig. Q4(a)

Obtain the element stresses of the stepped bar shown in Fig. Q4(b) using penalty approach. A₁ = 2400 mm² L₁ =150 mm; E₁ = 70 GPa; A₂ = 750 mm² L₂ = 300 mm E₂ = 200 GPa; $P = 200 \times 10^3 N$. (10 Marks)



PART - B

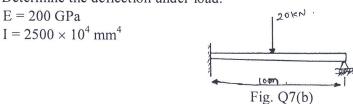
- 5 a. Explain briefly the iso-parametric, sub-parametric and super parametric elements, (06 Marks)
 - b. Derive the shape function of 2D quadrilateral element of linear model. (08 Marks)
 - c. Evaluate the following integral using two-point and 3-point gauss-integration method.

$$I = \int_{-1}^{+1} \left(3\xi^3 + 2\xi^2 + \xi + 2 \right) d\xi \quad . \tag{06 Marks}$$

- 6 a. Derive the stiffness matrix for a 1 D truss element. (08 Marks)
 - b. For the two bar truss shown in Fig. Q6(b) determine the nodal displacement. Take E = 200 GPa; $A_1 = A_2 = 200 \text{ mm}^2$. (12 Marks)



- 7 a. Derive Hermite shape function for beam element. (06 Marks)
 - A uniform C S beam is fixed at one end and supported by a roller at the other end.
 A concentrated load 20 kN is applied at the mid length of beam as shown in Fig. Q7(b).
 Determine the deflection under load.
 (14 Marks)



- 8 a. Discuss the Galerkin approach for 1 D heat conduction problem. (10 Marks)
 - b. Consider the brick wall of thickness L = 0.3 m, $k = 0.7 \text{ W/m}^{\circ}\text{C}$. The inner surface is at 28°C and outer surface is exposed to cold air at -15°C . Heat transfer coefficient on outer surface $h = 40 \text{ W/m}^{2}{}^{\circ}\text{C}$. Determine steady state temperature distribution with the wall and heat flux through the wall. Use two element model. (10 Marks)